mediately apparent that these formulas are useful for very small values of $k_0 b$. At a given value of $k_0 b$, whether Young's formula would be applicable or not can be determined from the percent deviation of the dc value from the true frequency dependent value of the capacitance $C_{1P} + C_{2P}/2$. For example, at $k_0 b = 0.8$, the deviation is 5.6 percent and 8.6 percent for $1/a = 0.25$ and 0.025, respectively.

Fig. 4 shows the experimental tuning characteristics of a long reentrant cavity [5]. The resonant frequency for a particular gap width was also calculated theoretically by treating the cavity as a coaxial line shorted at one end and terminated at the other by i) the short-circuit capacitance given by Marcuvitz's formula and by ii) the frequency dependent capacitance $2C_{1P} + C_{2P}$ obtained by the variational method. The theoretical curve obtained from the latter is identical in shape to the experimental curve though there is a quantitative disagreement, which is 2 percent at the worst. The curve obtained from using Marcuvitz's formula shows 2.5 percent deviation at the lowest end of the tuning range and the discrepancy increases rapidly. Since, for the dimensions of the experimental cavity, $k_0 b = 1.32$ at 7.0 GHz, this disagreement is not unexpected considering the frequency dependent nature of the terminating gap capacitance.

IV. Conclusions

A systematic theoretical study has been made on the equivalent circuit of a gap in the central conductor of a TEM coaxial line, using the standard variational technique. The computed parameter values show good agreement with the experimental data available in the literature. Further, experimental verification is provided by computing the resonant frequencies of a reentrant cavity for various gap widths from the short-circuit gap capacitances and comparing those with the experimentally measured values. As long as the normalized frequency $k_0 b \ll 2.405$ and the gap width is small, Young's or Marcuvitz's formula can predict the resonant frequency. But as the resonant frequency increases with increasing gap width, it is the frequency dependent behavior of the short-circuit terminating capacitance that plays the key role in predicting the tuning characteristics correctly.

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Analysis of Triangular Microstrip Resonators

ARVIND K. SHARMA, MEMBER, IEEE AND BHARATHI BHAT, SENIOR MEMBER, IEEE

***Abstract*—An isosceles triangular microstrip resonator is analyzed with the full wave formulation of the spectral domain technique. For a given apex angle and triangle height, the resonant frequency is evaluated from the numerical solution of the determinantal characteristic equation, obtained by neglecting the transverse current density. The agreement between the theoretical and experimental results is typically within ±2 percent.**

I. Introduction

The triangular microstrip resonator is a potential network element for a wide variety of applications such as oscillators, filters, and circulators [1]. In a recent investigation, Helszajn and James [2], and Nisbet and Helszajn [3] studied the equilateral triangular microstrip resonator element for filter and circulator applications. The 120° symmetry property of this element was utilized in an articulate design of circulator [2], [4]. Cuhaci and James [5] showed that, as a resonator, this element exhibits slightly higher radiation Q-factor (Q_r) than the corresponding circular microstrip disk resonator. This is a significant advantage in the design of low-loss microwave integrated circuits.

The isosceles triangular microstrip resonator, as shown in Fig. 1, is considered to be a useful network element, especially for oscillator and filter applications. It can provide greater flexibility compared with the equilateral configuration in the design of microwave integrated circuits.

In this paper, we present an analysis of the isosceles triangular microstrip resonator with the full wave formulation of the spectral domain technique. The experimental verification of the computed resonant frequencies for various apex angles and triangle heights is also included.

II. Analysis

The isosceles triangle element in a shielding waveguide configuration is shown in Fig. 1. It has an apex angle 2α and height l. The dielectric thickness d above the ground plane has relative dielectric constant ϵ_r. The shielding waveguide has dimension $2a$ and $d + h$. The triangular region is the surface bounded by lines given by the following equations:

$$\text{Triangular Region, } T: \begin{cases} z = l & \text{(1a)} \\ x \pm z \tan\alpha = 0. & \text{(1b)} \end{cases}$$

The spectral domain analysis of this structure is essentially similar to that of a rectangular microstrip resonator [6] or any other microstrip resonant structure [7]–[9]. Therefore, we shall present here a description of the assumed current density only.

The current density distribution on an isosceles triangular microstrip resonator is not explicitly known. However, as a first approximation, we neglect the transverse current density and assume the variation of the longitudinal current density $J_z(x,z)$ as following:

$$J_z(x,z) = J_z(x) J_z(z) \tag{2}$$

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A. K. Sharma is with the Microwave Technology Center, RCA Laboratories, David Sarnoff Research Center, Princeton, NJ 08540.
B. Bhat is with the Centre for Applied Research for Electronics, Department of Electrical Engineering, Indian Institute of Technology, New Delhi-110016, India.

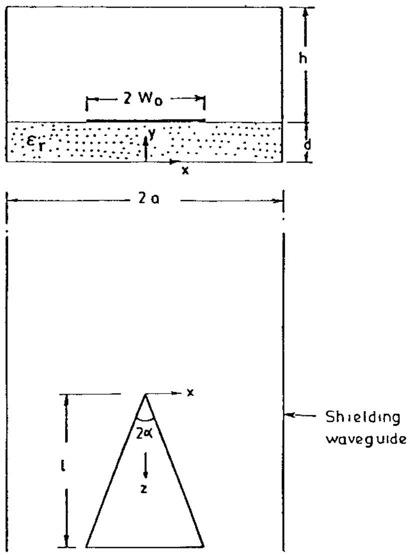

Fig. 1. Isosceles triangular microstrip resonator.

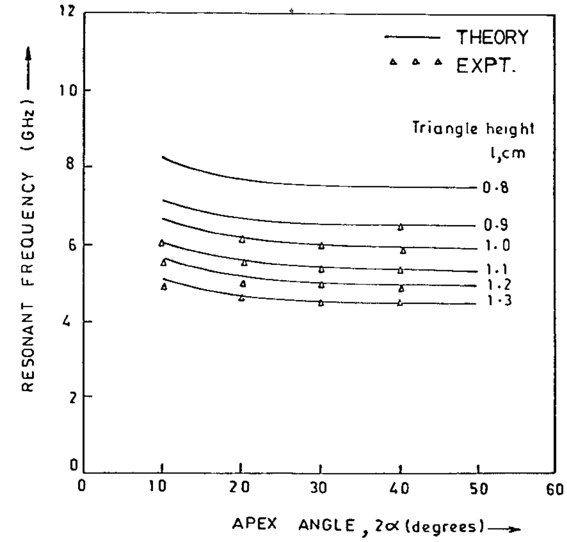

Fig. 2. Resonant frequency of an isosceles triangular microstrip resonator for various apex angles. $2a = 1.27$ cm, $d + h = 1.27$ cm, $d = 0.0635$ cm, $\epsilon_r = 10.2$.

where

$$J_z(x) = \begin{cases} 1 + \left| \dfrac{x}{w_0} \right|^3 & \text{on } T \\ 0 & \text{elsewhere} \end{cases} \tag{3}$$

and

$$J_z(z) = \begin{cases} \sin \dfrac{\pi z}{l} & \text{on } T \\ 0 & \text{elsewhere} \end{cases} \tag{4}$$

where $2w_0 = 2l \tan \alpha$ is the base of the triangle. The Fourier transform of $J_z(x, z)$ is found with the definition of discrete Fourier transform [6, eq. (2)].

It is pertinent to point out that in a physical situation, the transverse current density is not negligible at all. However, we consider the presence of only the longitudinal current density in this investigation. This approximation leads to considerable reduction in the mathematical complexity and in the computer time required to compute a resonant frequency.

III. Resonant Frequency

We have developed a computer program to evaluate the resonant frequency of an isosceles triangular microstrip resonator. This computer program was verified by comparing the results of a rectangular microstrip resonator with those reported by Itoh [6]. In the computations, the inner product is computed with an accuracy up to three significant digits or more. The root of the characteristic equation is also determined with the same accuracy. The computer time required for a typical computation is about 35 min on an ICL 1909 system (UK) which is about four to five times slower than an IBM 360 system.

The resonant frequency obtained with this technique is plotted in Fig. 2 for various apex angles between 10° and 50°. For the equilateral and the isosceles right-angled triangular resonator, it is plotted in Fig. 3. Several triangular resonators have been realized on Epsilum-10 ($\epsilon_r = 10.2$) substrates. In the measurement of their fundamental resonant frequency, the resonators were coupled to a 50-Ω microstrip line [3]. Coupling between the resonator and microstrip line was provided by a capacitive gap. The effect of the coupling on the resonant frequency was observed to be negligible. The resonant frequency was measured in

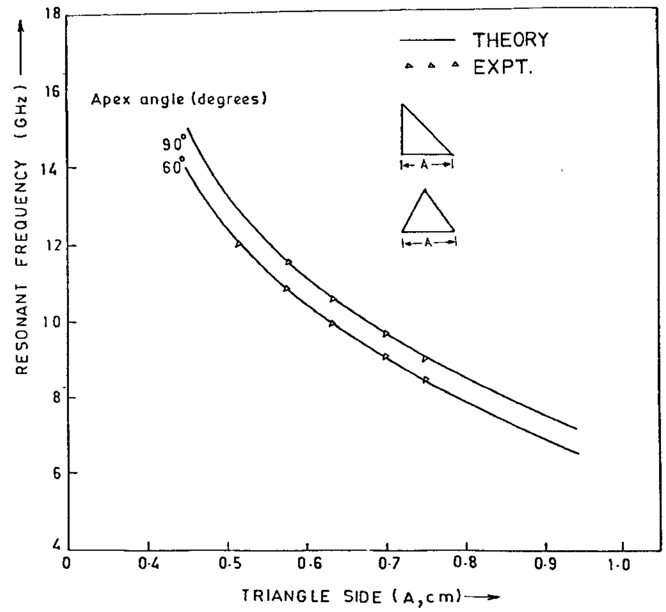

Fig. 3. Resonant frequency of equilateral and isosceles right-angled triangular microstrip resonators as a function of triangle side. $2a = 1.27$ cm, $d + h = 1.27$ cm, $d = 0.0635$ cm, $\epsilon_r = 10.2$.

the reflection mode. The experimental values were observed in the frequency bands of 3.2–6.5 GHz and 8.2–12.4 GHz. These values are shown in Figs. 2 and 3 in the background of theoretical results. The agreement between theoretical and experimental values is typically better than ±2 percent. The Q-factors of these resonators on Epsilum-10 substrate were observed to be in the range of 50 to 100.

IV. Conclusions

An analysis of the isosceles triangular microstrip resonator has been presented with the full wave formulation of the spectral domain technique. The resonant frequencies of the triangular resonators with various apex angles agree within ±2 percent of the computed results. Therefore, the assumed form of the longitudinal current density is adequate for all practical purposes and it is not necessary to take into account the transverse current density. The present analysis can be utilized to predict resonant frequencies of the isosceles triangular microstrip resonators with good accuracy.

The isosceles triangular microstrip resonator has a wide variety of applications in resonator and filter networks. The apex angle and triangle height provide additional flexibility in the design. They can also be used as a radiating element in various array applications.

The nonavailability of an adequate analysis and design data on triangular shape has so far precluded the possibility of their application as a network element at microwave frequencies. The theoretical data presented in this paper will provide a designer an insight into the controllability of various governing parameters.

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Magnetostatic Wave Dispersive Delay Line

PRADEEP WAHI AND ZYGMOND TURSKI, MEMBER, IEEE

Abstract —**The physical limits of the interdigital transducer (IDT) implementation for surface acoustic wave (SAW) devices beyond VHF/UHF have restricted their use in microwave systems. Inherent dispersion of magnetostatic wave (MSW) devices is suitable for signal processing directly at microwave frequencies. Dispersion enhancement tests performed on MSW delay lines confirm their suitability to an *S*-band simultaneous pulse separator for EW receiver applications.**

I. Introduction

Ability to produce frequency dependent phase delays and dispersion, in UHF and VHF bands, enabled the development of signal-processing components such as dispersive filters, delay lines, pulse compressors, pulse expanders, convolvers, and correlators [1], [2]. High dispersion in small physical dimensions is commonly achieved using SAW technology, where the acoustic-wave velocity is of the order of 10^5 cm/s. Upper frequency limitations stem from the photolithographic ability to resolve narrow line widths and gaps with nearly one hundred percent yield. The state-of-the-art devices operate up to 1.8 GHz and seemingly approach the upper frequency limit [3].

Since the magnetostatic wave propagation velocities are at least an order of magnitude higher than those of SAW, it should be possible to achieve much higher frequency operation maintaining the same line resolution. It is of great advantage, however, that MSW launching could be achieved through a single line geometry [4].

Intrinsically dispersive characteristics of MSW, although nonlinear, are of immediate use in EW applications, such as a simultaneous pulse separator [5]. It has been observed that nonchannelizeed receivers often read incorrect information in the presence of simultaneous signals [6]. The simultaneous signal separator can be used in conjunction with an inexpensive IFM receiver to improve the probability of intercept in a dense EW environment. For this purpose, a highly dispersive delay line is required to separate in time-domain simultaneous signals differing in frequency. Typically, simultaneous signals 100 MHz apart may require more than 30-ns time separation for individual detection.

Quasi-TEM bulk or planar structure provide insufficient dispersion sensitivity, dispersion per unit length, for realization in small size. MSW dispersion at microwave frequencies is ideally suited for this application. Additionally, the MSW dispersion characteristics can be linearized to obtain upchirp or downchirp, enabling extensive signals processing in microwave frequencies [7]. A brief summary of MSW modes is presented in Section II. Experimental results of coupling and dispersion in a MSSW device and the temperature effects on the delay characteristics are discussed in Section III. Section IV discusses dispersion modification techniques and is followed by a summary in Section V.

II. Magnetostatic Waves

Magnetostatic waves are inherently dispersive, magnetically dominated electromagnetic waves. The three fundamental wave categories are: 1) Magnetostatic Surface Waves (MSSW); 2) Magnetostatic Forward Volume Waves (MSFVW); and 3) Magnetostatic Backward Volume Waves (MSBVW). Recent development in liquid-phase epitaxial growth techniques for high-quality bubble memory YIG films on Gadolinium Gallium Garnet (GGG) substrates revived the interest in MSW for microwave applications. MSW devices can operate at frequencies in excess of 20 GHz with associated losses below 25 dB/μS. Associated dispersion and time delays of 100 ns/cm can be readily achieved.

The particular MSW wave that can exist in a YIG-film geometry is determined solely by the orientation of a bias field relative to the YIG film and propagation direction. When the bias field is normal to the YIG film, only MSFVW's can exist. When the bias field lies in the plane of the YIG film, MSBVW's exist for the film, direction parallel to the propagation vector $\boldsymbol{k}$ and MSSW exist for the field normal to the direction of propagation. Among the three modes, only MSBVW's exhibit negative dispersion, downchirp, characteristics (i.e., the phase and group velocities have contra-directed components along each other). The theoretical propagation bandwidth limits for the three modes are defined by k values of 0 and ∞. The frequency range for the volume waves is

$$f_0 < f < [f_0(f_0 + f_m)]^{1/2} \tag{1}$$

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The authors are with Litton Amecom, 5115 Calvert Rd., College Park, MD 20740.

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